Is Heavy Baryon Approach Necessary?

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Abstract

On the simple model of interacting massless pseudoscalar and heavy fermion fields it is demonstrated that using appropriately chosen renormalization condition one can respect power counting within the relativistic theory without applying the technique of heavy baryon approach. Explicit calculations are performed for diagrams including two-loops. It is argued that the introduction of the heavy baryon chiral perturbation theory was useful but unnecessary.

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I. INTRODUCTION

Chiral symmetry plays an important role in the low energy dynamics of strongly interacting particles. Using this symmetry Weinberg and Gasser and Leutwyller have developed a systematic rigorous scheme for processes involving only mesons, so-called chiral perturbation theory [1,2].

The nontrivial problem appeared after the work of Gasser, Sainio and Švarc who have considered processes with a single baryon [3]. They noticed that there is no consistent power counting when a baryon is included: higher order loops contribute in low order (in small expansion parameters) calculations. Performing any given order (in small expansion parameters) calculations one needs to include contributions of diagrams with an increasing (up to infinity) number of loops.

To avoid this problem, Jenkins and Manohar used the ideas of the heavy quark effective field theory and suggested to take extremally non-relativistic limit of the fully relativistic theory [4]. The expansion in inverse powers of baryon mass M allowed them to develop the so-called Heavy Baryon Chiral Perturbation Theory (HBCHPT) which avoids severe complications appearing in relativistic treatment of baryons at low energies, encountered in [3]. Nowadays HBCHPT is an important and effective method of calculation of different processes involving electro-magnetic and strong interactions (For a review and references see [5] and [6]).

Let us remind that problems of the relativistic approach that multi-loop diagrams contribute into low order calculations [3] were actually encountered in $\overline{MS}$ scheme. The $\overline{MS}$ scheme puts the effective cut-off equal to the largest involved mass scale i.e. nucleon mass and violates the power counting. One might ask if the violation of power counting is the intrinsic feature of theory of pion-nucleon interactions, or it is an artifact of the particular method of calculations only. Power counting certainly depends on applied subtraction scheme (renormalization condition).

For processes involving an arbitrary number of nucleons Weinberg suggested to use renormalization points of the order of external momenta or less [7,8]. Such schemes put the effective cutoff for loop diagrams of the order of external momenta and make power counting applicable for loop integrals. While Weinberg considered a non-relativistic effective field theory, the same idea of appropriate choice of renormalization condition can be useful in relativistic theory as well. As was argued in [9] those parts of relativistic diagrams which are responsible for the violation of power counting can be changed by adding counter-terms. Hence they can be removed by choosing appropriate renormalization condition.

In present paper we consider a model of interacting heavy fermion and massless scalar fields. This model is a simple analogue of relativistic chiral perturbation theory. Calculating one and two-loop Feynman diagrams we explicitly demonstrate the above statement that choosing appropriate subtraction scheme (renormalization condition) one can respect power counting in the relativistic theory without appealing to the heavy baryon technique. It has been argued by Lutz [10] that a consistent power counting can exist within relativistic scheme. Our discussion here is different from Lutz's approach.

II. ONE-LOOP ANALYSIS

We consider a field theoretical model described by the Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\phi\partial_\mu\partial^\mu\phi + \bar{\psi}\left(i\gamma^\mu\partial_\mu - M\right)\psi - g\bar{\psi}\gamma_5\gamma^\mu\psi\partial_\mu\phi + L_1 \tag{1}$$

In (1) ψ is a fermion field with mass M, ϕ is a neutral massless pseudoscalar field, g is a coupling constant dimension of which in units of mass is $(2-n)/2$ and throughout this paper we use dimensional regularization with n being the space-time dimension. L_1 is an infinite series containing all counter-terms which are necessary to remove all divergences.

Lagrangian (1) suggests that analogously to the meson chiral perturbation theory [1] for diagrams containing one fermion line we can have a naive(?) power counting. We assign +4 powers (of small momenta) to each loop integration, +1 power to each derivative occurring in the interaction term, -2 powers to each scalar propagator and -1 power to each fermion propagator. Thus to each particular diagram i it is assigned the resulting power N_i. We will say that diagram i obeys power counting if the leading term of the result of actual calculation depends on a small momentum k as $k^{N_i}f(k)$, where $f(k)$ is a constant or logarithmic function of k.

This power counting is badly violated if $\overline{MS}$ scheme is used: higher loops do not correspond to higher powers of k.

Below on the example of fermion propagator we demonstrate that using appropriately chosen renormalization condition it is possible to retain the power counting within relativistic theory.

Let us consider one-loop correction to the fermion propagator depicted in FIG.1 a) where the solid line corresponds to the fermion and dashed line corresponds to the pseudoscalar.

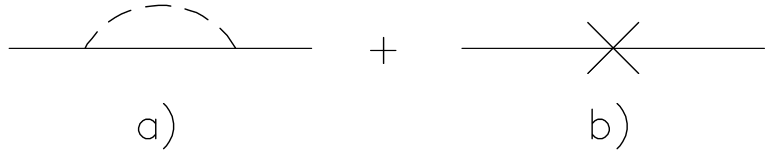

FIG. 1. *a) One-loop correction to the fermion propagator and b) corresponding counter-term diagram.*

According to our power counting the result of this diagram is expected to be of third order in small parameter. In the framework of dimensional regularization an expression for this diagram is given by the following integral:

$$\Sigma_1 = -\frac{g^2}{(2\pi)^n}\int \frac{d^n q\ \gamma_5\ \not{q}\,(\not{p}+\not{q}+M)\,\gamma_5\ \not{q}}{[q^2+i\epsilon]\,[(p+q)^2-M^2+i\epsilon]} \tag{2}$$

where $\not{p} = p^\mu\gamma_\mu$, $p^\mu = Mv^\mu + k^\mu$ is off mass-shell momenta of the fermion, $v_\mu v^\mu = 1$ and $k^\mu(<< M)$ is a small quantity. (2) can be reduced to the following form:

$$\Sigma_1 = g^2 M J(01) - g^2(n-1)J^{n+2}(11)\ \not{p} \tag{3}$$

where $J(01)$ and $J^{n+2}(11)$ are given in the Appendix.

Substituting the values of $J(01)$ and $J^{n+2}(11)$ into (3) we obtain:

$$\begin{aligned}\Sigma_1 = & -M\frac{ig^2}{(4\pi)^{n/2}}\left(M^2\right)^{n/2-1}\Gamma\left(1-n/2\right) - \not{p}\frac{3}{2}\frac{ig^2}{(4\pi)^{n/2}}\left(M^2\right)^{n/2-1}\Gamma\left(1-n/2\right)\\ & - \not{p}p^2\frac{ig^2}{(4\pi)^{n/2}}\left(M^2\right)^{n/2-2}\frac{\Gamma\left(2-n/2\right)}{n-2} - \not{p}\frac{ig^2}{(4\pi)^{n/2}}\left(M^2\right)^{n/2-1}\frac{\Gamma\left(3-n/2\right)}{(2-n)(3-n)}\left(1-z\right)^2\\ & - \not{p}\frac{ig^2}{(4\pi)^{n/2}}\left(M^2\right)^{n/2-1}\frac{\Gamma\left(4-n/2\right)}{(2-n)(3-n)(4-n)}\left(1-z\right)^3\times {}_2F_1\left(1,4-n/2;5-n;1-z\right)\\ & + \not{p}\frac{ig^2}{(4\pi)^{n/2}}\left(M^2\right)^{n/2-1}\Gamma\left(n/2\right)\Gamma\left(2-n\right)\left(1-z\right)^{n-1}\times {}_2F_1\left(n/2,n;n;1-z\right)\end{aligned} \tag{4}$$

where $z \equiv p^2/M^2$ and ${}_2F_1$ is the Gauss hypergeometric function [11,12].

To carry out the renormalization procedure we need to add contributions of counter-terms to (4). As it was mentioned above all necessary counter-terms are included in the Lagrangian and the corresponding contribution reads:

$$\delta\Sigma_1 = g^2\delta_1 + g^2\delta_2\ \not{p} + g^2\delta_3 p^2\ \not{p} + g^2\delta_4 p^4\ \not{p} \tag{5}$$

Equation (4) fixes only the divergent parts of δ_i which have to be the same for every scheme and the choice of the particular renormalization scheme is equivalent to the choice of finite

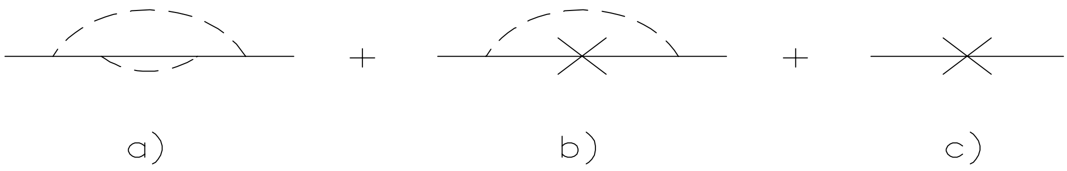

FIG. 2. *a) Two-loop correction to the fermion propagator, b) corresponding diagram with one loop sub-diagram replaced with g^2-order counter-terms and c) g^4-order counter-term diagram.*

parts of counter-terms. Note that δ_4 is finite - we are free to add such finite counterterms, it is our choice of renormalization scheme.

We choose δ_4 and the finite parts of δ_1, δ_2 and δ_3 so as to exactly cancel first four terms in (4). The remaining expression admits the limit $n \to 4$ and we obtain for renormalized self-energy:

$$\Sigma_1^R = -\frac{ig^2}{32\pi^2} M^2 \not p \left[\frac{\left(M^2 - p^2\right)^3}{M^2 p^4} \ln\left(1 - p^2/M^2\right) + \frac{\left(M^2 - p^2\right)^3}{M^4 p^2} \right] \tag{6}$$

Note that $p^2 = M^2 + 2M v_\mu k_\mu + k^2$ and thus (6) is in agreement with power counting - $\Sigma_1^R \sim k^3$.

On the other hand applying $\overline{MS}$ to (4) and taking into account the mass and wave function renormalizations we get $\Sigma^R_{1(\overline{MS})} \sim (M^2 - p^2) \sim k$.

III. TWO-LOOP ANALYSIS

Two-loop corrections to the fermion propagator have more complicated structure. For our illustrating purposes it is enough to consider a diagram depicted in FIG.2 a). It leads to the following expression:

$$\Sigma_2 = \frac{ig^4}{(2\pi)^{2n}} \int \frac{d^n q_1 d^n q_2}{\left[q_1^2 + i\epsilon\right]\left[q_2^2 + i\epsilon\right]} \gamma_5 \not q_1 S_F\left(p + q_1\right) \gamma_5 \not q_2 S_F\left(p + q_1 + q_2\right) \gamma_5 \not q_2 S_F\left(p + q_1\right) \gamma_5 \not q_1 \tag{7}$$

Simplifying (7) we obtain:

$$\begin{aligned}\Sigma_2 = ig^4 \Big\{ &\left(M^2 - p^2\right) \gamma^\mu J_\mu(1101) - 2 \not p \left[\frac{M^2 - p^2}{2n} J(1101) - \frac{2}{n} p_\mu J^\mu(1101) + \left(1 - \frac{1}{n}\right) p^2 C_2 \right] \\ &+ 2M J(01)J(01) - 4M^2 \gamma^\mu J_\mu(11) J(01) + 4M^2 (\not p + M) J(01) J(02) \\ &+ 4M^2 \left(p^2 - M^2\right) \gamma^\mu J_\mu(12) J(01) \Big\}\end{aligned} \tag{8}$$

where $J_\mu(1101)$, $J(1101)$, C_2, $J_\mu(11)$, $J(02)$ and $J_\mu(12)$ are given in the appendix.

To renormalize diagram a) in FIG.2 consistently we need to add diagrams b) and c) in FIG.2. Diagram b) is obtained by substituting one loop sub-diagram of diagram a) by g^2-order counterterms. Diagram c) is a contribution of g^4-order counterterms. Calculating diagram b) in FIG.2 and adding to (8) we obtain:

$$\tilde{\Sigma}_2 = ig^4\left\{\left(M^2-p^2\right)\gamma^\mu J_\mu(1101) - 2\not{p}\left[\frac{M^2-p^2}{2n}J(1101) - \frac{2}{n}p_\mu J^\mu(1101) + \left(1-\frac{1}{n}\right)p^2 C_2\right]\right\} \tag{9}$$

Contribution of g^4 order counter-terms originated by diagram c) in FIG.2 is:

$$\delta\Sigma_2 = g^4\delta_5\not{p} + g^4\delta_6 p^2\not{p} + g^4\delta_7 p^4\not{p} + g^4\delta_8 p^6\not{p} + g^4\delta_9 p^8\not{p} \tag{10}$$

Again we added terms with finite δ_8 and δ_9 which are fixed below by the renormalization condition together with finite parts of δ_5, δ_6 and δ_7.

Next step is to rewrite (9) in terms of $(1-z)$, expand analytic (in $(1-z)$) part in powers of $(1-z)$ and add $\delta\Sigma_2$ expressed in terms of z and M^2:

$$\delta\Sigma_2 = \not{p}M^2 g^4\left\{\delta_5/M^2 + \delta_6 + M^2\delta_7 + M^4\delta_8 + M^6\delta_9 - (1-z)\left[\delta_6 + 2M^2\delta_7 + 3M^4\delta_8 + 4M^6\delta_9\right]\right.$$
$$\left. +(1-z)^2\left[M^2\delta_7 + 3M^4\delta_8 + 6M^6\delta_9\right] - (1-z)^3\left[M^4\delta_8 + 4M^6\delta_9\right] + (1-z)^4 M^6\delta_9\right\} \tag{11}$$

δ_8 and δ_9 and finite parts of δ_5, δ_6 and δ_7 are fixed so as to exactly cancel the first five terms (up to and including $(1-z)^4$) in the expansion of the analytic part. Performing these calculations we get:

$$\Sigma_2^R = ig^4\not{p}\frac{(M^2)^{n-2}}{(4\pi)^n}\left\{-\frac{3}{4}\left[\mathrm{Li}_2(1-z) - (1-z) - \frac{(1-z)^2}{4}\right] + \frac{z(1-z)^3}{12} - \frac{(1-z)^4}{48z} + \frac{29}{192}(1-z)^4\right.$$
$$\left. + \frac{(1-z)^5}{12z} + \ln(1-z)\left[-\frac{9}{8}(1-z)^2 - \frac{3}{4}\ln z - \frac{3}{4}z(1-z) - \frac{(1-z)^3}{4z} + \frac{(1-z)^4}{16z^2}\right]\right\} \tag{12}$$

In (12) Li_2 is the dilogarithm function [12]. It is straightforward to check that the coefficient function in front of $\ln(1-z)$ and the analytic part are both $O\left((1-z)^5\right)$ in (12). Hence "naive" power counting is again respected by the renormalized expression (12).

On the other hand applying $\overline{MS}$ scheme and taking into account the mass and wave function renormalization we obtain $\Sigma^R_{2(\overline{MS})} \sim (M^2-p^2) \sim k$. Hence two-loop diagram contributes at the same order as the one loop diagram as was observed in [3].

Let us summarise in short our strategy of dealing with the relativistic chiral perturbation theory.

To remove divergences from Feynman diagrams one can use the forest formula of Zimmermann. It is applied to individual diagrams and subtracts overall divergence and subdivergences. These subtractions can be implemented as actual counterterms in the Lagrangian [13]. For practical purposes one does not necessarily need to explecetely write down counterterms. We can specify the subtraction scheme (give a recipe for fixing finite parts) and consider parameters of the Lagrangian as finite renormalised coupling constants. In case of relativistic chiral perturbation theory instead of widely used $\overline{MS}$ scheme we should use a subtraction scheme which respects power counting. In particular, first we renormalise one loop diagrams by expanding analytic (in small momenta) parts in powers of small momenta and subtracting first few terms so as to respect the power counting. According [9] the non-analytic parts automatically respect power counting. For two-loop diagrams we first subtract one-loop subdiagrams and after expand analytic parts in powers of small momenta and subtract first few terms so as to respect the power counting. The non-analytic parts automatically respect power counting when subdiagrams are already subtracted. For three-loop diagrams we first subtract one and two-loop subdiagrams and after act analogously to the two-loop case etc. This iterative procedure is analogous and well defined for any number of loops. Since within the suggested subtraction scheme higher order diagrams do not contribute into lower order calculations, low order couplings do not need to be redifined when the results of higher order calculations are taken into account.

IV. CONCLUSIONS

In this work we have demonstrated that the problems of the relativistic chiral perturbation theory, in particular that multi-loop diagrams contribute into low order calculations encountered in [3], can be solved within relativistic approach using appropriately chosen renormalization condition. Hence it is unnecessary to use heavy baryon approach. While the last approach simplifies calculations for many physical quantities, for the scalar form-factor of the nucleon it leads to the series which is not convergent near threshold [5] (Note that this problem has been successfully resolved by Becher and Leutwyler using "infrared regularization" [14]). The original relativistic approach never encounters this problem. Hence both approaches enjoy their advantages and have full right to exist.

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APPENDIX A

$$z \equiv \frac{p^2}{M^2}$$

$$J(01) = \frac{1}{(2\pi)^n}\int \frac{d^n q}{\left[q^2 - M^2 + i\epsilon\right]} = \frac{-i}{(4\pi)^{n/2}}\left(M^2\right)^{n/2-1}\Gamma\left(1-n/2\right) \tag{A1}$$

$$J^{n+2}(11) = \frac{2}{(2\pi)^{n+1}}\int \frac{d^{n+2} q}{\left[q^2 + i\epsilon\right]\left[(p+q)^2 - M^2 + i\epsilon\right]} = \frac{i}{(4\pi)^{n/2}}\left(M^2\right)^{n/2-1}\left\{\frac{\Gamma\left(1-n/2\right)\Gamma\left(n-1\right)}{\Gamma\left(n\right)}\times\right.$$

$$
{}_2F_1\left(1,1-n/2;2-n;1-z\right)+\Gamma\left(n/2\right)\Gamma\left(1-n\right)\left(1-z\right)^{n-1}{}_2F_1\left(n/2,n;n;1-z\right)\Bigg\} \tag{A2}
$$

$$
J(11)=\frac{1}{(2\pi)^n}\int\frac{d^nq}{\left[q^2+i\epsilon\right]\left[(p+q)^2-M^2+i\epsilon\right]}=\frac{i\left(M^2\right)^{n/2-2}}{(4\pi)^{n/2}}\Bigg\{\frac{\Gamma\left(2-n/2\right)\Gamma\left(n-3\right)}{\Gamma\left(n-2\right)}\times
$$

$$
{}_2F_1\left(1,2-n/2;4-n;1-z\right)+\Gamma\left(n/2-1\right)\Gamma\left(3-n\right)\left(1-z\right)^{n-3}{}_2F_1\left(n/2-1,n-2;n-2;1-z\right)\Bigg\} \tag{A3}
$$

$$
\begin{aligned}
J(1101)&=\frac{1}{(2\pi)^{2n}}\int\frac{d^nq_1d^nq_2}{\left[q_1^2+i\epsilon\right]\left[q_2^2+i\epsilon\right]\left[(p+q_1+q_2)^2-M^2+i\epsilon\right]}\\
&=\frac{\left(M^2\right)^{n-3}}{(4\pi)^n}\Bigg\{\frac{\Gamma\left(3-n\right)\Gamma^2\left(n/2-1\right)\Gamma\left(2-n/2\right)}{\Gamma\left(n/2\right)}+\frac{\Gamma\left(4-n\right)\Gamma^2\left(n/2-1\right)\Gamma\left(3-n/2\right)}{\Gamma\left(n/2+1\right)}z\\
&+\frac{1}{12}\left[6-2\pi^2+15z+12\mathrm{Li}_2(1-z)+6\ln(1-z)\left(\frac{(1-z)^2}{z}+2(1-z)+2\ln z\right)\right]\Bigg\}
\end{aligned} \tag{A4}
$$

$$
\begin{aligned}
J^\mu(1101)&=\frac{1}{(2\pi)^{2n}}\int\frac{d^nq_1d^nq_2\ q_1^\mu}{\left[q_1^2+i\epsilon\right]\left[q_2^2+i\epsilon\right]\left[(p+q_1+q_2)^2-M^2+i\epsilon\right]}\\
&=-p^\mu\frac{\left(M^2\right)^{n-3}}{(4\pi)^n}\Bigg\{\frac{\Gamma\left(3-n\right)\Gamma\left(n/2-1\right)\Gamma\left(2-n/2\right)\Gamma\left(n/2\right)}{\Gamma\left(n/2+1\right)}\\
&+\frac{\Gamma\left(4-n\right)\Gamma\left(n/2-1\right)\Gamma\left(3-n/2\right)\Gamma\left(n/2\right)}{\Gamma\left(n/2+2\right)}z+\frac{1}{24}\left[\frac{61}{3}-2\pi^2-\frac{40}{3}(1-z)-2(1-z)^2-2(1-z)^3\right.\\
&\left.-2\frac{(1-z)^4}{z}+12\mathrm{Li}_2(1-z)+\ln(1-z)\left(12[1-z+\ln z]+\frac{6(1-z)^2}{z}-\frac{2(1-z)^3}{z^2}\right)\right]\Bigg\}
\end{aligned} \tag{A5}
$$

$$
J^{\mu\nu}(1101)=\frac{1}{(2\pi)^{2n}}\int\frac{d^nq_1d^nq_2\ q_1^\mu\ q_2^\nu}{\left[q_1^2+i\epsilon\right]\left[q_2^2+i\epsilon\right]\left[(p+q_1+q_2)^2-M^2+i\epsilon\right]}=C_1g^{\mu\nu}+C_2p^\mu p^\nu \tag{A6}
$$

$$
\begin{aligned}
C_2&=\frac{\left(M^2\right)^{n-3}}{(4\pi)^n}\Bigg\{\frac{\Gamma\left(3-n\right)\Gamma\left(2-n/2\right)\Gamma^2\left(n/2\right)}{\Gamma\left(n/2+2\right)}+\frac{\Gamma\left(4-n\right)\Gamma\left(3-n/2\right)\Gamma^2\left(n/2\right)}{\Gamma\left(n/2+3\right)}z\\
&+\frac{1}{72}\left[\frac{241}{12}-2\pi^2-\frac{5}{2}(1-z)-\frac{5}{2}(1-z)^2-\frac{3}{2}(1-z)^3-\frac{5}{2}\frac{(1-z)^4}{z}+\frac{(1-z)^4(1+z)}{z^2}\right.\\
&\left.+12\mathrm{Li}_2(1-z)+\ln(1-z)\left(12[1-z+\ln z]+\frac{6(1-z)^2}{z}-\frac{2(1-z)^3}{z^2}+\frac{(1-z)^4}{z^3}\right)\right]\Bigg\}
\end{aligned} \tag{A7}
$$

REFERENCES

[1] S.Weinberg, Physica A97 (1979) 327;

[2] J.Gasser and H.Leutwyler, Ann. Phys. (N.Y.) 158 (1984) 142; Nucl. Phys. B250 (1985) 465, 517, 539;

[3] J.Gasser, M.E.Sainio and Švarc, Nucl.Phys. **B307**,779 (1988);

[4] E.Jenkins and A.V.Manohar, Phys.Lett. **B 255**, 558 (1991);

[5] V.Bernard, N.Kaiser, Ulf-G.Meissner, Int.J.Mod.Phys. E4 (1995) 193-346, hep-ph/9501384;

[6] N.Fettes, Ulf-G. Meissner, S.Steininger Nucl.Phys.A640:199-234,1998, hep-ph/9803266;

[7] S.Weinberg, Phys.Lett. **B251** (1990) 288;

[8] S.Weinberg, Nucl.Phys. **B363** (1991) 3;

[9] J.Gegelia and G.Japaridze, hep-ph/9908377, to appear in Phys. Rev. **D**;

[10] M.Lutz, in proceedings of the workshop "The Standard Model at Low Energies" held at ECT* in Trento, Italy, from April 29 to May 10, 1996, hep-ph/9606301; Nucl. Phys. **A642** (1998) 171;
nucl-th/9906028;

[11] A. Prudnikov, Yu. Brichkov, O. Marichev, *Integrals and Series vol.3*, "Nauka", Moscow (1986);

[12] Handbook of Mathematical Functions. Ed. by M.Abramowitz and I.A.Stegun, Dover Publications, NY (1972);

[13] John C. Collins, Renormalization, Cambridge University Press, Cambridge (1984);

[14] T. Becher and H. Leutwyler, hep-ph/9901384;